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# Chiral perturbation theory vs. vector meson dominance in the decays $\phi \to \rho\gamma\gamma$ and $\phi \to \omega\gamma\gamma$

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## Abstract

It is pointed out that the radiative decays of a $\phi$ meson, $\phi \to \rho\gamma\gamma$ and $\phi \to \omega\gamma\gamma$, receive dominant contributions from the pseudoscalar ($P = \eta, \eta^{'}$) exchanges. Using the vector meson dominance model, we find that $B(\phi \to \rho\gamma\gamma) \approx 1.3 \times 10^{-4}$ and $B(\phi \to \omega\gamma\gamma) \approx 1.5 \times 10^{-5}$, which are mainly from the $\eta^{'}$ pole. Thus, these decays are well within the reach of the $\phi$ factory. Our estimates are a few orders of magnitude larger than the chiral loop contributions in the heavy vector meson chiral lagrangian, which is about ( a few )

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$\times 10^{-9}$.

1. Radiative decays of $\phi$ mesons, $\phi \to \rho\gamma\gamma$ and $\phi \to \omega\gamma\gamma$, are rare processes, and thus have never been observed yet. However, these may be observed in the $\phi$ factory at Frascati where $10^{11}$ $\phi$'s would be produced per year. In this vein, it would be interesting to estimate the branching ratios for these decays in a reasonable way.

In a different context, Leibovich *et al.* have recently considered these decays in the heavy vector meson chiral lagrangian approach [1]. In this framework, there are two contributions to these processes : the loop (pseudoscalar-vector meson loop) and the anomaly term from $\phi \to \rho(\omega) + \pi^0$ followed by $\pi^0 \to \gamma\gamma$. The loop contribution depends on a parameter $g_2$ (the $VVP$ coupling) which enters in the heavy vector meson chiral lagrangian [1] :

$$B(\phi \to \rho\gamma\gamma)_{loop} = 5.8 \times 10^{-9} \left(\frac{g_2}{0.75}\right)^4, \tag{1}$$

$$B(\phi \to \omega\gamma\gamma)_{loop} = 4.2 \times 10^{-9} \left(\frac{g_2}{0.75}\right)^4, \tag{2}$$

where the results are normalized for $g_2 = 0.75$ as predicted in the chiral quark model. For $\phi \to \omega\gamma\gamma$, the anomaly contribution from $\phi \to \omega\pi^0$ is negligible compared to the loop contribution shown above. On the other hand, the anomaly term is not negligible for the other decay $\phi \to \rho\gamma\gamma$. However, one can still find a region in the phase space where the loop contributions dominate the anomaly contributions. Since the loop contributions to $\phi \to \rho\gamma\gamma$ and $\phi \to \omega\gamma\gamma$ are finite to the order considered in Ref. [1], the authors of Ref. [1] suggested that one may relate the measured branching ratios of $\phi \to \rho\gamma\gamma$ and $\phi \to \omega\gamma\gamma$ with Eqs. (1) and (2) in order to fix the low energy constant $g_2$ in the heavy meson chiral lagrangian.

In this letter, we point out that there is an important class of Feynman diagrams (see Fig. 1) which has been neglected in Ref. [1]. Feynman diagrams shown in Fig. 1 cannot occur in the heavy vector meson chiral lagrangian, since the heavy vector meson number is to be conserved in the chiral lagrangian approach [2] (in the absence of weak interactions). However, a vector meson can be either created or destroyed via electromagnetic interaction (*e.g.* $\omega \to \pi^0 + \gamma$, and $\eta' \to \rho^0 + \gamma$, etc.), unlike the heavy quarks or the heavy baryons whose numbers do not change by electromagnetic interactions. Therefore, there is no apparent reason why Fig. 1 can be ignored in $\phi \to \rho\gamma\gamma$ and $\phi \to \omega\gamma\gamma$, and it is the purpose of this

letter to calculate the contributions of Feynman diagrams shown in Fig. 1 in the usual vector meson dominance model. In fact, it has long been known that the similar diagrams are the most important in the decay $\eta \to \pi^0\gamma\gamma$ through the $\rho$ and $\omega$ exchanges [3].

2. The $VP\gamma$ vertex can be obtained from the following phenomenological lagrangian (which can be also derived from the chiral lagrangian with vector mesons in the hidden symmetry scheme [4]) :

$$\mathcal{L}(VP\gamma) = \frac{e}{g}\, g_{\omega\rho\pi}\, \epsilon_{\mu\nu\alpha\beta}\, F^{\mu\nu}\, \mathrm{Tr}\, \left[P\left\{Q, \partial^\alpha V^\beta\right\}\right], \tag{3}$$

where $g = g_{\rho\pi\pi} = 5.85$ using the KSFR relation, and

$$g_{\omega\rho\pi} = -\frac{3g^2}{8\pi^2 f_\pi}, \tag{4}$$

with $f_\pi = 93$ MeV being the pion decay constant. The matrix $Q = \mathrm{diag}(2/3, -1/3, -1/3)$ is the electric charges of three light quarks, and $P$ and $V$ are the $3 \times 3$ matrix fields for pseudoscalar and vector meson nonets :

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & K^0 \\ K^- & \bar{K^0} & -\frac{2}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 \end{pmatrix}, \tag{5}$$

$$V = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}}(\rho^0 + \omega) & \rho^+ & K^{*+} \\ \rho^- & \frac{1}{\sqrt{2}}(-\rho^0 + \omega) & K^{*0} \\ K^{*-} & \bar{K}^{*0} & -\phi \end{pmatrix}. \tag{6}$$

We assume the ideal mixing for $\omega$ and $\phi$, and a partial mixing for $\eta$ and $\eta'$ :

$$\eta = \eta_8\, \cos\theta_p - \eta_0\, \sin\theta_p, \tag{7}$$

$$\eta' = \eta_8\, \sin\theta_p + \eta_0\, \cos\theta_p, \tag{8}$$

with $\theta_p \simeq -20^\circ$.

One can extract the $VP\gamma$ ($V = \phi, \rho, \omega$ and $P = \pi^0, \eta, \eta'$) vertices from the above effective lagrangian, defining $C_{PV}$ as follows :

$$\mathcal{M}_{VP\gamma} = \frac{e}{g} g_{\omega\rho\pi} C_{PV} \epsilon_{\mu\nu\alpha\beta} k^{\alpha} \epsilon^{\beta}(\gamma) p^{\mu} \epsilon^{\nu}(V) \tag{9}$$

where $k$ and $p$ are the momenta of $\gamma$ and $V$ respectively. Explicit values of the coefficients $C_{PV}$'s are given in Table I.

Let us first check how good this $SU(3)_V$ symmetric interaction lagrangian is by considering various decays, $V \to P\gamma$ and $P \to V\gamma$. The decay rate for $i \to f + \gamma$ described by the above vertex is given by

$$\Gamma(i \to f + \gamma) = \frac{1}{(2S_i + 1)} \, \frac{g^2}{4\pi} \, C_{PV}^2 \, \frac{9 \; \alpha}{128\pi^3 f_\pi^2} \, M_i^3 \, \left[ 1 - \frac{M_f^2}{M_i^2} \right]^3 , \tag{10}$$

where $S_i$ is the spin of the initial particle, $i$.

In Table II, we present our predictions (using the $C_{PV}$'s in Table 1) along with the measured branching ratios. The agreements between the two are reasonably good, except for $\phi \to \pi^0\gamma$, which is OZI-forbidden decay and thus is of higher order in $1/N_c$. Also, note that our prediction, $B(\phi \to \eta^{'}\gamma) = 2.1 \times 10^{-4}$ is a factor of two below the current upper limit ($< 4.1 \times 10^{-4}$). Thus, this decay may be observed soon at the $\phi$ factory. Since $B(\eta^{'} \to \rho\gamma) = (30.2 \pm 1.3)\%$, this decay ($\phi \to \eta^{'}\gamma$) followed by $\eta^{'} \to \rho\gamma$ can constitute a large component of $\phi \to \rho\gamma\gamma$. The results shown in Table I suggest that the $VP\gamma$ interaction lagrangian, Eq. (3), may be used in studying other processes such as $\phi \to \rho\gamma\gamma$ and $\phi \to \omega\gamma\gamma$.

3. Using the $VP\gamma$ vertices in Table I, it is straightforward to calculate Feynman diagrams shown in Fig. 1, and get the decay rates and the $\gamma\gamma$ spectra in $\phi \to \rho\gamma\gamma$ and $\phi \to \omega\gamma\gamma$. It is convenient to use the following variables :

$$s \equiv (P_\phi - P_\rho)^2 = (k + k^{'})^2, \tag{11}$$

$$t \equiv (P_\phi - k)^2 = (P_\rho + k^{'})^2, \tag{12}$$

$$u \equiv (P_\phi - k^{'})^2 = (P_\rho + k)^2, \tag{13}$$

with $s + t + u = M_\phi^2 + M_\rho^2$. The allowed ranges for $s$ and $t$ are

$$0 \le s \le (M_\phi - M_\rho)^2, \qquad\qquad t_0 \le t \le t_1, \tag{14}$$

$$t_{0,1} = \frac{1}{2}\ \left[\left(M_\phi^2 + M_\rho^2 - s\right) \mp \sqrt{(M_\phi^2 + M_\rho^2 - s)^2 - 4M_\rho^2 M_\phi^2}\ \right]. \tag{15}$$

In terms of $s, t, u$ variables, the amplitude for $\phi \to \rho\gamma\gamma$ is given by

$$\begin{aligned}\mathcal{M} = &-(\frac{e}{g} g_{\omega\rho\pi})^2 \sum_{P=\eta,\eta'} c_{PV_1} c_{PV_2} \left(\frac{\epsilon_{\mu\alpha\kappa\sigma}\epsilon_{\nu\beta\lambda\tau}}{t - m_P^2 + i\ m_P \Gamma_P} + \frac{\epsilon_{\mu\beta\lambda\sigma}\epsilon_{\nu\alpha\kappa\tau}}{u - m_P^2 + i\ m_P \Gamma_P}\right) \\ &\times k_1^\kappa\ k_2^\lambda\ p_1^\sigma\ p_2^\tau\ \varepsilon^{*\mu}(k_1)\varepsilon^{*\nu}(k_2)\varepsilon^\alpha(p_1)\varepsilon^{*\beta}(p_2) \qquad (16) \\ = &-(f_t \epsilon_{\mu\alpha\kappa\sigma}\epsilon_{\nu\beta\lambda\tau} + f_u \epsilon_{\mu\beta\lambda\sigma}\epsilon_{\nu\alpha\kappa\tau})\ k_1^\kappa\ k_2^\lambda\ p_1^\sigma\ p_2^\tau\ \varepsilon^{*\mu}(k_1)\varepsilon^{*\nu}(k_2)\varepsilon^\alpha(p_1)\varepsilon^{*\beta}(p_2) \qquad (17)\end{aligned}$$

where

$$\begin{aligned} f_t &= (\frac{e}{g} g_{\omega\rho\pi})^2 \sum_{P=\eta,\eta'} \frac{c_{PV_1} c_{PV_2}}{t - m_P^2 + i\ m_P \Gamma_P} \\ f_u &= (\frac{e}{g} g_{\omega\rho\pi})^2 \sum_{P=\eta,\eta'} \frac{c_{PV_1} c_{PV_2}}{u - m_P^2 + i\ m_P \Gamma_P} \end{aligned} \tag{18}$$

where $V_1 = \phi$ and $V_2 = \rho($ or $\omega)$. For $P = \eta^{'}$, the intermediate propagator can develop a pole for $t$ (or $u$) $= m_{\eta'}^2$, and we have to include its decay width in the denominator of the propagator for $\eta^{'}$ : $\Gamma_{\eta'} = (0.198 \pm 0.019)$MeV.

The square of the above amplitude, when averaged over the initial spin, and summed over the final spins, is simplified as follows :

$$\begin{aligned}\overline{|\mathcal{M}|^2} &= \frac{1}{2} \cdot \frac{1}{3} \sum_{\text{spin}} |\mathcal{M}|^2 \\ &= \frac{1}{24} \Big[\ |f_t|^2 (t - m_1^2)^2 (t - m_2^2)^2 + |f_u|^2 (u - m_1^2)^2 (u - m_2^2)^2 \\ &\qquad + \mathrm{Re}(f_t f_u^*)(s^2 m_1^2 m_2^2 + (m_1^2 m_2^2 - tu)^2)\ \Big]. \end{aligned} \tag{19}$$

Here we have included the factor of $1/2$ in order to take into account two identical particles (two photons) in the final state. The decay rate can be obtained by integrating the following expression over the variable $t$ :

$$\frac{d\Gamma}{dm_{\gamma\gamma}^2} = \frac{1}{(2\pi)^3}\ \frac{1}{32 M_\phi^3}\ \int_{t_0}^{t_1}\ \overline{|\mathcal{M}|^2}\ dt. \tag{20}$$

After numerical integrations, we get

$$B(\phi \to \rho\gamma\gamma) = 1.3 \times 10^{-4}, \tag{21}$$

$$B(\phi \to \omega\gamma\gamma) = 1.5 \times 10^{-5}. \tag{22}$$

These are much larger compared to the loop contributions, Eqs. (1) and (2), obtained in the framework of the heavy vector meson chiral lagrangian approach [1]. Hence, the claim that these decays might be useful in constraining the coupling $g_2$ in the heavy vector meson chiral lagrangian may not be viable [1].

One can also study the $m_{\gamma\gamma}$ spectra in $\phi \to \rho\gamma\gamma$ and $\phi \to \omega\gamma\gamma$ from Eq. (20) as a function of $m_{\gamma\gamma}$, as shown in Fig. 2. The two spectra are the same except for (i) the overall normalization of a factor 1/9 from different Clebsch-Gordan coefficients (as shown in Table I), and (ii) the slight mass difference between $\rho$ and $\omega$. Both decays are dominated by $\phi \to \eta^{'}\gamma$ followed by $\eta^{'} \to \rho(\text{or } \omega)\gamma$.

4. In summary, we have considered the radiative decays of $\phi$ mesons, $\phi \to \rho\gamma\gamma$ and $\phi \to \omega\gamma\gamma$ in the vector meson dominance model. From the usual $VP\gamma$ vertices, we find that these decays occur at the level of $1.3 \times 10^{-4}$ and $1.5 \times 10^{-5}$ in the branching ratios, respectively, and thus should be within the reach of the $\phi$ factory at Frascati. Also, these contributions are dominant in size over the loop contributions considered in the framework of the heavy vector meson chiral perturbation theory [1], and pose a doubt on the claim made in Ref. [1]. Finally, our model predicts that $B(\phi \to \eta^{'}\gamma) = 2.1 \times 10^{-4}$, which is just a factor of two below the current upper limit and thus can be easily tested in the near future. This decay actually dominates $\phi \to \rho\gamma\gamma$ and $\phi \to \omega\gamma\gamma$ in our model. Thus, detection of this decay at the predicted level would constitute another test of our model based on the vector meson dominance.

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[1] There are also contributions from the anomaly via $\phi \to \rho(\text{or } \omega) + \pi^0(\text{or } \eta, \eta^{'})$ followed by $\pi^0, \eta, \eta^{'} \to \gamma\gamma$. These decays are dominated by the $\pi^0$ contribution, and can be suitably removed by imposing a cut on $m_{\gamma\gamma} \approx m_{\pi^0}$. Hence, we do not consider these possibilities in this work.

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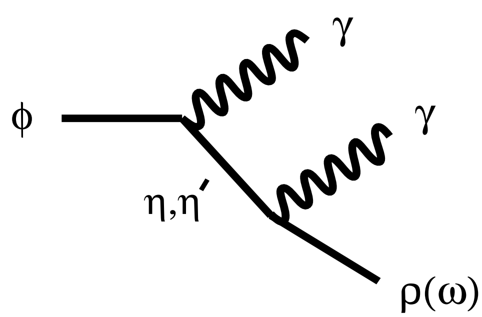


FIG. 1. Feynman diagrams for $\phi \to \rho\gamma\gamma$ and $\phi \to \omega\gamma\gamma$ in the vector meson dominance model.

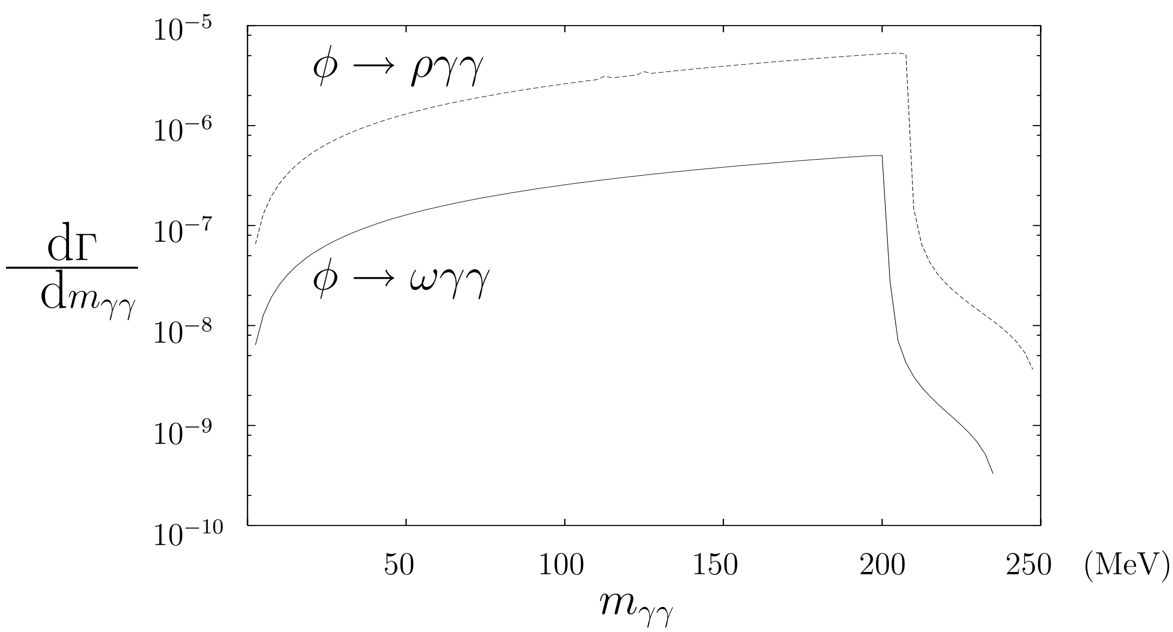


FIG. 2. The $m_{\gamma\gamma}$ spectra in $\phi \to \rho\gamma\gamma$ (the dashed curve) and $\phi \to \omega\gamma\gamma$ (the solid curve).

# TABLES

TABLE I. $C_{PV}$ defined in Eq. (7) for $P=\pi^0,\eta,\eta^{'}$ and $V=\rho^0,\omega,\phi$.

| $C_{PV}$ | $\pi$ | $\eta$ | $\eta'$ |
|---|---|---|---|
| $\rho$ | $-\frac{1}{3}$ | $\frac{1}{\sqrt{3}}(\sqrt{2}\sin\theta-\cos\theta)$ | $-\frac{1}{\sqrt{3}}(\sqrt{2}\cos\theta+\sin\theta)$ |
| $\phi$ | $0$ | $\frac{2}{3\sqrt{3}}(\sqrt{2}\cos\theta+\sin\theta)$ | $\frac{2}{3\sqrt{3}}(\sqrt{2}\sin\theta-\cos\theta)$ |
| $\omega$ | $-1$ | $\frac{1}{3\sqrt{3}}(\sqrt{2}\sin\theta-\cos\theta)$ | $-\frac{1}{3\sqrt{3}}(\sqrt{2}\cos\theta+\sin\theta)$ |

TABLE II. Branching ratios for $V\to P\gamma$ and $P\to V\gamma$ with $P=\pi^0,\eta,\eta^{'}$ and $V=\rho^0,\omega,\phi$.

| Decay Modes | Predictions | Data |
|---|---|---|
| $\omega\to\pi^0\gamma$ | 9.0% | $(8.5\pm0.5)\%$ |
| $\omega\to\eta\gamma$ | $9.5\times10^{-4}$ | $(8.3\pm2.1)\times10^{-4}$ |
| $\rho^0\to\pi^0\gamma$ | $5.3\times10^{-4}$ | $(7.9\pm2.0)\times10^{-4}$ |
| $\rho^0\to\eta\gamma$ | $4.1\times10^{-4}$ | $(3.8\pm0.7)\times10^{-4}$ |
| $\eta^{'}\to\rho^0\gamma$ | 34.3% | $(30.2\pm1.3)\%$ |
| $\eta^{'}\to\omega\gamma$ | 3.1% | $(3.02\pm0.30)\%$ |
| $\phi\to\pi^0\gamma$ | 0.0 | $(1.31\pm0.13)\times10^{-3}$ |
| $\phi\to\eta\gamma$ | 2.2% | $(1.28\pm0.06)\%$ |
| $\phi\to\eta^{'}\gamma$ | $2.1\times10^{-4}$ | $<4.1\times10^{-4}$ |